# J. K. SHAH CLASSES

## SYJC - MATHS & STATISTICS

**PRELIMINARY TEST - 2**

**Branch - Andheri, Borivali & Vasai** **Date: 25 /01/2017**

**Total Marks : 80** **Total time: 3 hours**

**SECTION - I**

**Q1. (A)Attempt any six of the following** **(12)**

**01.** Find X and Y if $X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ; $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

SOLUTION

$$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \qquad X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \qquad X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$2X = \begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \qquad 2Y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 10 & 0 \\ 2 & 8 \end{bmatrix} \qquad Y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix} \qquad Y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

**02.** find $\frac{dy}{dx}$ if $y = \sin^{-1}\sqrt{1 - x^2}$

SOLUTION

Put $x = \cos\theta$

$$y = \sin^{-1}\sqrt{1 - \cos^2\theta}$$

$$y = \sin^{-1}\sqrt{\sin^2\theta}$$

$$y = \sin^{-1}(\sin\theta)$$

$$y = \theta$$

$$y = \cos^{-1} x$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1 - x^2}}$$

**03.** Find the value of k if the function

$$f(x) = \frac{\tan 7x}{2x} \quad ; \quad x \neq 0$$

$$= k \quad ; \quad x = 0 \quad \text{is continuous at } x = 0$$

SOLUTION

**Step 1**

$\lim_{x \to 0} f(x)$

$$= \lim_{x \to 0} \frac{\tan 7x}{2x}$$

$$= \lim_{x \to 0} \frac{7}{2} \frac{\tan 7x}{7x}$$

$$= \frac{7}{2}(1)$$

$$= \frac{7}{2}$$

**Step 2 :**

f(0) = k ……… given

**Step 3 :**

Since $f$ is continuous at x = 0

$$f(0) = \lim_{x \to 0} f(x)$$

$$k = 7/2$$

**04.** Write negations of the following statements

1. $\forall\, y \in N$ , $y^2 + 3 \leq 7$

   **Negation** : $\exists\, y \in N$ , such that $y^2 + 3 > 7$

2. if the lines are parallel then their slopes are equal

   **Using** : $\sim(P \rightarrow Q) \equiv P \wedge \sim Q$

   **Negation** : lines are parallel and their slopes are not equal

**05.** find elasticity of demand if the marginal revenue is Rs 50 and the price is Rs 75

**SOLUTION**

$$Rm = R_A \left(1 - \frac{1}{\eta}\right)$$

$$50 = 75 \left(1 - \frac{1}{\eta}\right)$$

$$\frac{50}{75} = 1 - \frac{1}{\eta}$$

$$\frac{2}{3} = 1 - \frac{1}{\eta}$$

$$\frac{1}{\eta} = 1 - \frac{2}{3}$$

$$\frac{1}{\eta} = \frac{1}{3} \qquad \eta = 3$$

**06.** State which of the following sentences are statements . In case of statement , write down the truth value

a) Every quadratic equation has only real roots

ans : the given sentence is a logical statement . Truth value : F

b) $\sqrt{-4}$ is a rational number

ans : the given sentence is a logical statement . Truth value : F

**07.** Evaluate : $\int \frac{\sec^2 x}{\tan^2 x + 4} dx$

SOLUTION PUT $\tan x = t$

$$\sec^2 x . dx = dt$$

THE SUM IS

$$= \int \frac{1}{t^2 + 4} dt$$

$$= \int \frac{1}{t^2 + 2^2} dt$$

$$= \frac{1}{a} \tan^{-1} \frac{t}{a} + c$$

$$= \frac{1}{2} \tan^{-1} \frac{t}{2} + c$$

Resubs.

$$= \frac{1}{2} \tan^{-1}\left(\frac{\tan x}{2}\right) + c$$

**08.** if $A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ ; $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find $|AB|$

SOLUTION

AB

$= \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1 + 3 & 2 + 4 \\ 2 + 6 & 4 + 8 \end{bmatrix}$

$= \begin{bmatrix} 4 & 6 \\ 8 & 12 \end{bmatrix}$

$|AB| = 4(12) - 8(6) = 48 - 48 = 0$

**Q2. (A)Attempt any TWO of the following** **(06)**

**01.** $f(x) = \dfrac{3 - \sqrt{2x + 7}}{x - 1}$ ; $x \neq 1$

$= -1/3$ ; $x = 1$ Discuss continuity at x = 1

SOLUTION

**STEP 1**

$\lim\limits_{x \to 1} f(x)$

$= \lim\limits_{x \to 1} \dfrac{3 - \sqrt{2x+7}}{x - 1}$

$= \lim\limits_{x \to 1} \dfrac{3 - \sqrt{2x+7}}{x - 1} \cdot \dfrac{3 + \sqrt{2x+7}}{3 + \sqrt{2x+7}}$

$= \lim\limits_{x \to 1} \dfrac{9 - (2x+7)}{x - 1} \cdot \dfrac{1}{3 + \sqrt{2x+7}}$

$= \lim\limits_{x \to 1} \dfrac{9 - 2x - 7}{x - 1} \cdot \dfrac{1}{3 + \sqrt{2x+7}}$

$= \lim\limits_{x \to 1} \dfrac{2 - 2x}{x - 1} \cdot \dfrac{1}{3 + \sqrt{2x+7}}$

$= \lim\limits_{x \to 1} \dfrac{2(1 - x)}{x - 1} \cdot \dfrac{1}{3 + \sqrt{2x+7}}$

$= \lim\limits_{x \to 1} \dfrac{-2(x - 1)}{x - 1} \cdot \dfrac{1}{3 + \sqrt{2x+7}}$ $\quad x - 1 \neq 0$

$= \dfrac{-2}{3 + \sqrt{2 + 7}}$

$$= \frac{-2}{3+3}$$

$$= \frac{-1}{3}$$

**STEP 2 :**

$f(1) = -1/3$ ……… given

**STEP 3 :**

$f(1) = \lim_{x \to 1} f(x)$ ; $f$ is continuous at x = 1

**02.** Write the converse , inverse and the contrapositive of the statement

"if two triangles are not congruent then their areas are not equal"

SOLUTION :

**LET** $P \to Q$ ≡ if the triangles are not congruent then their areas are not equal

**CONVERSE** : $Q \to P$

If the areas of triangles are not equal then they are not congruent

**CONTRAPOSITIVE :** $\sim Q \to \sim P$

If the areas of the triangles are equal then they are congruent

**INVERSE** : $\sim P \to \sim Q$

If the two triangles are congruent then their areas are equal

**03.** Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{6x}{1-5x^2}\right)$

SOLUTION

$$y = \tan^{-1}\left(\frac{5x + x}{1 - 5x.x}\right)$$

$$y = \tan^{-1} 5x + \tan^{-1} x$$

$$\frac{dy}{dx} = \frac{1}{1+25x^2} . \frac{d}{dx}(5x) + \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{5}{1+25x^2} + \frac{1}{1+x^2}$$

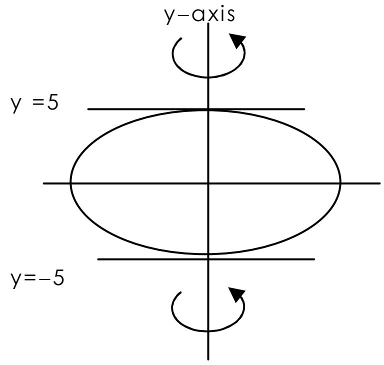
**(B) Attempt any TWO of the following** **(08)**

**01.** Find the volume of a solid obtained by the complete revolution of the ellipse

$$\frac{x^2}{36} + \frac{y^2}{25} = 1 \quad \text{about Y – axis}$$

SOLUTION

**STEP 1 :**




$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a^2 = 36$ ; $a = 6$

$b^2 = 25$ , $b = 5$

**STEP 2 :**

$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$

$$\frac{x^2}{36} = 1 - \frac{y^2}{25}$$

$$\frac{x^2}{36} = \frac{25 - y^2}{25}$$

$$x^2 = \frac{36}{25}(25 - y^2)$$

**STEP 3 :**

$$V = \pi \int_{-5}^{5} x^2.dy$$

About y – axis

$$= \pi \int_{-5}^{5} \frac{36}{25}(25 - y^2).dy$$

$$= \frac{36\pi}{25} \int_{-5}^{5} (25 - y^2).dy$$

$$= \frac{36\pi}{25}\left[25y - \frac{y^3}{3}\right]_{-5}^{5}$$

$$= \frac{36\pi}{25}\left\{\left(125 - \frac{125}{3}\right) - \left(-125 + \frac{125}{3}\right)\right\}$$

$$= \frac{36\pi}{25}\left\{\left(\frac{375 - 125}{3}\right) - \left(\frac{-375 + 125}{3}\right)\right\}$$

$$= \frac{36\pi}{25}\left\{\left(\frac{250}{3}\right) - \left(\frac{-250}{3}\right)\right\}$$

$$= \frac{36\pi}{25}\left(\frac{500}{3}\right)$$

$= 240\,\pi$ cubic units

**02.** Evaluate : $\int \log(1 + x^2)\, dx$

$$= \int \log(1 + x^2).\, 1dx$$

$$= \log(1 + x^2) \int 1\, dx - \int\left(\frac{d}{dx}\log(1 + x^2) \int 1\, dx\right) dx$$

$$= \log(1 + x^2)\,.\, x - \int \frac{2x}{1 + x^2.}\,.\, x \quad dx$$

$$= x.\log(1 + x^2) - 2\int \frac{x^2}{1 + x^2.} \quad dx$$

$$= x.\log(1 + x^2) - 2\int \frac{1 + x^2 - 1}{1 + x^2.}\,.\, dx$$

$$= x.\log(1 + x^2) - 2\int 1 - \frac{1}{1 + x^2.} \quad dx$$

$$= x.\log(1 + x^2) - 2\left(x - \tan^{-1}x\right) + c$$

$$= x.\log(1 + x^2) - 2x + 2\tan^{-1}x + c$$

**03.** If Mr. Rao orders x cupboards , with demand function as

$$p = 2x + \frac{32}{x^2} - \frac{5}{x}$$

How many cupboards should he order for the most economical deal

**SOLUTION**

**STEP 1 : COST**

$$C = p.x$$

$$= \left(2x + \frac{32}{x^2} - \frac{5}{x}\right) . x$$

$$= 2x^2 + \frac{32}{x} - 5$$

**STEP 2 :**

$$\frac{dC}{dx} = 4x - \frac{32}{x^2} = 4x - 32x^{-2}$$

$$\frac{d^2C}{dx^2} = 4 + 64x^{-3}$$

$$= 4 + \frac{64}{x^3}.$$

**STEP 3 :**

$$\frac{dC}{dx} = 0$$

$$4x - \frac{32}{x^2} = 0$$

$$4x = \frac{32}{x^2}$$

$$4x^3 = 32$$

$$x^3 = 8 \qquad \therefore\ x = 2$$

**STEP 4 :**

$$\left.\frac{d^2C}{dx^2}\right|_{x = 2} = 4 + \frac{64}{2^3} > 0$$

Cost is minimum at x = 2

Mr. Rao must order 2 cupboards

**Q3.** **(A)Attempt any TWO of the following** **(06)**

**01.** Using the truth table , examine whether the statement pattern is a tautology , a contradiction or a contingency : $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$

SOLUTION

| p | q | ~ p | p → q | ~p ∨ q | (p → q) ↔ (~p ∨ q) |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | F | F | F | T |
| F | T | T | T | T | T |
| F | F | T | T | T | T |

Since all values in the last column are 'T ' , the given statement is a TAUTOLOGY

**02.** $f(x) = \dfrac{(e^{3x} - 1)^2}{x.\log(1 + 3x)}$ ; $x \neq 0$

$= 10$ ; $x = 0$ Discuss the continuity at x = 0

**Solution :**

**Step 1**

$\lim_{x \to 0} f(x)$

$= \lim_{x \to 0} \dfrac{(e^{3x} - 1)^2}{x.\log(1 + 3x)}$

Dividing Numerator & Denominator by $x^2$

$x \rightarrow 0$ , $x \neq 0$ , $x^2 \neq 0$

$$= \lim_{x \to 0} \frac{\dfrac{(e^{3x} - 1)^2}{x^2}}{\dfrac{x.\log(1 + 3x)}{x^2}}$$

$$= \lim_{x \to 0} \frac{\left(\dfrac{e^{3x} - 1}{x}\right)^2}{\dfrac{\log(1 + 3x)}{x}}$$

$$= \lim_{x \to 0} \frac{\left(3\ \dfrac{e^{3x} - 1}{3x}\right)^2}{\log(1 + 3x)\ \dfrac{1}{x}}$$

$$= \lim_{x \to 0} \frac{\left(3 \cdot \frac{e^{3x} - 1}{3x}\right)^2}{\log\left((1 + 3x)^{\frac{1}{3x}}\right)^3}$$

$$= \frac{(3.\log e)^2}{\log e^3}$$

$$= \frac{9}{3.\log e} = 3$$

**Step 2 :**

$f(0) = 10$ ……… given

**Step 3 :**

$$f(0) \neq \lim_{x \to 0} f(x)$$

$\therefore f$ is discontinuous at $x = 0$

**Step 4 :**

**Removable Discontinuity**

$f$ can be made continuous at $x = 0$ by redefining it as

$$f(x) = \frac{(e^{3x} - 1)^2}{x.\log(1 + 3x)} \quad ; \quad x \neq 0$$

$$= 3 \quad ; \quad x = 0$$

**03.** if $\sin y = x.\sin(5 + y)$ ; prove that $\dfrac{dy}{dx} = \dfrac{\sin^2(5 + y)}{\sin a}$

SOLUTION

$$\sin y = x.\sin(5 + y)$$

$$x = \frac{\sin y}{\sin(5 + y)}$$

Differentiating wrt y

$$\frac{dx}{dy} = \frac{\sin(5 + y)\frac{d}{dy}\sin y - \sin y\frac{d}{dy}\sin(5 + y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin(5 + y) . \cos y - \sin y . \cos(5 + y)\frac{d}{dy}(5 + y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin(5 + y).\cos y - \cos(5 + y) . \sin y}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin(5 + y - y)}{\sin^2(5 + y)}$$

$$\frac{dx}{dy} = \frac{\sin 5}{\sin^2(5 + y)}$$

Now $\dfrac{dy}{dx} = \dfrac{1}{\frac{dx}{dy}}$

$\therefore$ $\dfrac{dy}{dx} = \dfrac{\sin^2(5+ y)}{\sin 5}$

**(B) Attempt any TWO of the following** **(08)**

**01.** $$\int_4^7 \frac{(11-x)^2}{x^2 + (11-x)^2}\, dx \quad \text{.....} \quad \textbf{(1)}$$

**USING** $\int_a^b f(x)dx = \int_b^b f(a+b-x)\, dx$

$$I = \int_4^7 \frac{\left[11 - (4+7-x)\right]^2}{(4+7-x)^2 + \left[11 - (4+7-x)\right]^2}\, dx$$

$$I = \int_4^7 \frac{\left[11 - (11-x)\right]^2}{(11-x)^2 + \left[11 - (11-x)\right]^2}\, dx$$

$$I = \int_4^7 \frac{(11-11+x)^2}{(11-x)^2 + (11-11+x)^2}\, dx$$

$$I = \int_4^7 \frac{x^2}{(11-x)^2 + x^2}\, dx \quad \text{.....} \quad \textbf{(2)}$$

(1) + (2)

$$2I = \int_4^7 \frac{(11-x)^2 + x^2}{(11-x)^2 + x^2}\, dx$$

$$2I = \int_4^7 1 \, dx$$

$$2I = \left[x\right]_4^7$$

$$2I = 7 - 4$$

$$2I = 3$$

$$I = 3/2$$

**02.** $$\int \frac{x^2}{x^4 + 5x^2 + 6}\, dx$$

$$\int \frac{x^2}{(x^2 + 2)(x^2 + 3)}\, dx$$

**SOLUTION**

$$\frac{x^2}{(x^2 + 2)(x^2 + 3)} = \frac{A}{x^2 + 2} + \frac{B}{x^2 + 3}$$

**$x^2$ = t (say)**

$$\frac{t}{(t + 2)(t + 3)} = \frac{A}{t + 2} + \frac{B}{t + 3}$$

$$t = A(t + 3) + B(t + 2)$$

**Put $t = -3$**

$$-3 = B(-3 + 2)$$

$$-3 = B(-1) \qquad \therefore\ B = 3$$

**Put $t = -2$**

$$-2 = A(-2 + 3)$$

$$-2 = A(1) \qquad \therefore\ A = -2$$

THEREFORE

$$\frac{t}{(t + 2)(t + 3)} = \frac{-2}{t + 2} + \frac{3}{t + 3}$$

HENCE

$$\frac{x^2}{(x^2 + 2)(x^2 + 3)} = \frac{-2}{x^2 + 2} + \frac{3}{x^2 + 3}$$

BACK IN THE SUM

$$= \int \frac{-2}{x^2 + 2} + \frac{3}{x^2 + 3}\, dx$$

$$= \int \frac{-2}{x^2 + \sqrt{2}^2} + \frac{3}{x^2 + \sqrt{3}^2}\, dx$$

$$= -2 \cdot \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + 3\ \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

$$= -\sqrt{2} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + \sqrt{3} \tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$

**03.** $A = \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$ Verify : $A.(\text{adj } A) = (\text{adj } A).A = |A|.I$

**COFACTOR'S**

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 1(0-0) = 0$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -1(9+2) = -11$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 1(0-0) = 0$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -1(-3-0) = 3$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1(3-2) = 1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1(0+1) = -1$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 1(2-0) = 2$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -1(-2-6) = 8$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 1(0+3) = 3$$

**COFACTOR MATRIX OF A**

$$\begin{pmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{pmatrix}$$

**ADJ A** **=** TRANSPOSE OF THE COFACTOR MATRIX

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

**|A|**

$= 1(0 + 0) + 1(9 + 2) + 2(0 - 0)$

$= 11$

**LHS 1**

$=$ A.(adj A)

$$= \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix} \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 11 + 0 & 3 - 1 - 2 & 2 - 8 + 6 \\ 0 - 0 - 0 & 9 + 0 + 2 & 6 + 0 - 6 \\ 0 - 0 + 0 & 3 + 0 - 3 & 2 + 0 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

**LHS 2**

$=$ (adj A) . A

$$= \begin{pmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{pmatrix}$$

$$= \begin{pmatrix} 0 + 9 + 2 & 0 + 0 + 0 & 0 - 6 + 6 \\ -11 + 3 + 8 & 11 + 0 + 0 & -22 - 2 + 24 \\ 0 - 3 + 3 & 0 - 0 + 0 & 0 + 2 + 9 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

**RHS**

$=$ |A|.I

$$= 11 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{pmatrix}$$

HENCE A.(adj A) = (adj A).A = |A|.I

**SECTION – II**

**Q4. (A)Attempt any six of the following (12)**

**01.** Find correlation coefficient between x and y for the following data

$n = 100$, $\bar{x} = 62$, $\bar{y} = 53$, $\sigma x = 10$, $\sigma y = 12$, $\Sigma(x - \bar{x})(y - \bar{y}) = 8000$

SOLUTION

$$r = \frac{cov(x,y)}{\sigma x \cdot \sigma y}$$

$$= \frac{\frac{\Sigma(x - \bar{x})(y - \bar{y})}{n}}{\sigma x \cdot \sigma y}$$

$$= \frac{\frac{8000}{100}}{10.12}$$

$$= \frac{80}{10.12}$$

$$= \frac{2}{3}$$

**02.** a building is insured for 80% of its value . The annual premium at 70 paise percent amounts to Rs 2,800 . Fire damaged the building to the extent of 60% of its value . How much amount for damage can be claimed under the policy

SOLUTION

Property value = · x

Insured value $= \frac{80x}{100} = \frac{4x}{5}$

Rate of premium = 70 paise percent

= 0.70%

Premium = · 2800

$2800 = \frac{0.70}{100} \times \frac{4x}{5}$

$2800 = \frac{7}{1000} \times \frac{4x}{5}$

$2800 = \frac{28x}{5000}$

$x = 100 \times 5000$

$x = 5,00,000$

Property value = · 5,00,000

Loss $= \frac{60}{100} \times 5,00,000$

$= \cdot\ 3,00,000$

Claim $= 80\%$ of loss

$= \frac{80}{100} \times 3,00,000$

$= \cdot\ 2,40,000$

**03.** The coefficient of rank correlation for a certain group of data is 0.5 . If $\Sigma d^2 = 42$ , assuming no ranks are repeated ; find the no. of pairs of observation

SOLUTION

$R = 0.5 \quad ; \quad \Sigma d^2 = 42$

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$0.5 = 1 - \frac{6(42)}{n(n^2 - 1)}$$

$$\frac{6(42)}{n(n^2 - 1)} = 1 - 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = 0.5$$

$$\frac{6(42)}{n(n^2 - 1)} = \frac{1}{2}$$

$$n(n^2 - 1) = 6 \times 42 \times 2$$

$$n(n^2 - 1) = 3 \times 2 \times 3 \times 2 \times 7 \times 2$$

$$(n - 1).n.(n + 1) = 7 \times 8 \times 9$$

On comparing , $n = 8$

**04.** Maya and Jaya started a business by investing equal amount . After 8 months Jaya withdrew her amount and Priya entered the business with same amount of capital . At the end of the year there was a profit of · 13,200 . Find their share of profit

SOLUTION

**STEP 1 :**

Profits will be shared in the

**'RATIO OF PERIOD OF INVESTMENT'**

| | MAYA | | JAYA | | PRIYA |
|---|---|---|---|---|---|
| = | 12 | : | 8 | : | 4 |

TOTAL = 24

**STEP 2 :**

PROFIT = · 13,200

Maya's share of profit = $\frac{12}{\cancel{24}_{\,2}} \times \cancel{13,200}^{\,1100}$ = · 6,600

Jaya's share of profit = $\frac{8}{\cancel{24}_{\,2}} \times \cancel{13,200}^{\,1100}$ = · 4,400

Priya'sshare of profit = $\frac{4}{\cancel{24}_{\,2}} \times \cancel{13,200}^{\,1100}$ = · 2,200

**05.** Calculate CDR for district A and B and compare

| Age Group (Years) | DISTRICT A | | DISTRICT B | |
|---|---|---|---|---|
| | NO. OF PERSONS **P** | NO. OF DEATHS **D** | NO. OF PERSONS **P** | NO. OF DEATHS **D** |
| 0 – 10 | 1000 | 18 | 3000 | 70 |
| 10 – 55 | 3000 | 32 | 7000 | 50 |
| Above 55 | 2000 | 41 | 1000 | 24 |
| | **$\Sigma$P = 6000** | **$\Sigma$D = 91** | **$\Sigma$P = 11000** | **$\Sigma$D = 144** |

CDR(A) = $\frac{\Sigma D}{\Sigma P} \times 1000$

= $\frac{91}{6000} \times 1000$

= 15.17

(DEATHS PER THOUSAND)

CDR(B) = $\frac{\Sigma D}{\Sigma P} \times 1000$

= $\frac{144}{11000} \times 1000$

= 13.09

(DEATHS PER THOUSAND)

COMMENT : CDR(B) < CDR(A) . HENCE DISTRICT B IS HEALTHIER THAN DISTRICT A

**06.** the probability distribution function of continuous random variable X is given by

$$f(x) = \frac{x}{4}, \quad 0 < x < 2$$

$$= 0, \quad \text{otherwise} \qquad \text{Find } P(x \le 1)$$

SOLUTION

the pdf of continuous random variable X is given by

$$f(x) = \frac{x}{4} \quad ; \quad 0 < x < 2$$

$$= 0 \quad ; \quad \text{otherwise}$$

$$P(x \le 1) = \int_0^1 \frac{x}{4}\, dx$$

$$= \left[\frac{x^2}{8}\right]_0^1$$

$$= \left(\frac{1}{8}\right) - \left(\frac{0}{8}\right)$$

$$= \frac{1}{8}$$

**07.** The ratio of incomes of Salim & Javed was 20:11 . Three years later income of Salim has increased by 20% and income of Javed was increased by Rs 500 . Now the ratio of their incomes become 3 : 2 . Find original incomes of Salim and Javed

**SOLUTION**

Let income of Salim = 20x

Income of Javed = 11x

As per the given condition

$$\frac{20x + \frac{20}{100}(20x)}{11x + 500} = \frac{3}{2}$$

$$\frac{20x + 4x}{11x + 500} = \frac{3}{2}$$

$$\frac{24x}{11x + 500} = \frac{3}{2}$$

$$48x = 33x + 1500$$

$$15x = 1500$$

$$x = 100$$

$\therefore$

Salim's original income = 20(100) = · 2000

Javed's original income = 11(100) = · 1100

**08.** for an immediate annuity paid for 3 years with interest compounded at 10% p.a. its present value is Rs 10,000 . What is the accumulated value after 3 years ( $1.1^3 = 1.331$)

**SOLUTION**

$$A = P(1 + i)^n$$

$$= 10000(1 + 0.1)^3$$

$$= 10000(1.1)^3$$

$$= 10000(1.331)$$

$$= \cdot\ 13,310$$

**Q5. (A)Attempt any Two of the following** **(06)**

**01.** a new treatment for baldness is known to be effective in 70% of the cases treated . Four bald members from different families are treated . Find the probability that

(i) at least one member is successfully treated (ii) exactly 2 members are successfully treated

**SOLUTION**

4 bald members from different families are treated , n = 4

For a trial Success – a defective pen

p – probability of success = 70/100 = 7/10

q – probability of failure = 1 – 7/10 = 3/10

r.v. X – no of successes = 0 , 1 , 2 , 3 , 4 **X ~ B (4 , 7/10)**

a) P(at least one member is successfully treated)

$= P(X \geq 1)$

$= P(1) + P(2) + \ldots\ldots + P(4)$

$= 1 - P(0)$

$= 1 - {}^4C_0 \cdot p^0 \cdot q^4$

$= 1 - {}^4C_0 \left(\frac{7}{10}\right)^0 \left(\frac{3}{10}\right)^4$

$= 1 - \dfrac{1 \cdot 1 \cdot 81}{10000}$

$= 1 - 0.0081$

$= 0.9919$

b) P(exactly 2 members are successfully treated)

$= P(X = 2)$

$= {}^4C_2 \cdot p^2 \cdot q^2$

$= {}^4C_2 \left(\frac{7}{10}\right)^2 \left(\frac{3}{10}\right)^2$

$= \dfrac{6 \cdot 49 \cdot 9}{10^4}$

$= \dfrac{2646}{10000} = 0.2646$

**02.** Compute rank correlation coefficient for the following data

| | | | | | | |
|---|---|---|---|---|---|---|
| Rx : | 1 | 2 | 3 | 4 | 5 | 6 |
| Ry : | 6 | 3 | 2 | 1 | 4 | 5 |

**SOLUTION**

| x | y | d = \|x – y\| | $d^2$ |
|---|---|---|---|
| 1 | 6 | 5 | 25 |
| 2 | 3 | 1 | 1 |
| 3 | 2 | 1 | 1 |
| 4 | 1 | 3 | 9 |
| 5 | 4 | 1 | 1 |
| 6 | 5 | 1 | 1 |
| | | | $\Sigma d^2 = 38$ |

$$R = 1 - \frac{6\Sigma d^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6(38)}{6(36 - 1)}$$

$$= 1 - \frac{38}{35}$$

$$= \frac{-3}{35}$$

$$= -0.086$$

**03.** the income of the agent remains unchanged though the rate of commission is increased from 5% to 6.25% . Find the percentage reduction in the value of the business

**SOLUTION**

Let initial sales = · 100

Rate of commission = 5%

∴ Commission = · 5

Let the new sales = · x

Rate of commission = 6.25%

∴ Commission = $\frac{6.25x}{100}$

Since the income of the broker remains unchanged

$$\frac{6.25\,x}{100} = 5$$

$$x = \frac{5 \times 100 \text{ X } 100}{625}$$

$$x = 80$$

∴ new sales = · 80

Hence percentage reduction in the value of the business = 20%

**(B) Attempt any Two of the following** **(08)**

**01.** A warehouse valued at · 10,000 contained goods worth · 60,000 . The warehouse was insured against fire for · 4,000 and the goods to the extent of 90% of their value . A fire broke out and goods worth · 20,000 were completely destroyed , while the remainder was damaged and reduced to 80% of its value . The damage to the warehouse was to the extent of · 2,000 . Find the total amount that can be claimed

**SOLUTION :**

**WAREHOUSE**

Property value = · 10,000

Insured value = · 4,000

Loss = · 2,000

Claim $= \frac{\text{insured val.}}{\text{Property val.}} \times \text{loss}$

$= \frac{4,000}{10,000} \times 2,000$

= · 800

**STOCK IN WAREHOUSE**

Value of stock = · 60,000

Insured value = 90% of the stock

Loss

Note : Since the remainder was reduced to 80% of its value the loss on it is 20%

$= 20,000 + \frac{20}{100}(60,000 - 20,000)$

$= 20,000 + \frac{20}{100}(40,000)$

= 20, 000 + 8,000

= · 28,000

Since 90% of the stock was insured

Claim = 90% of loss

$= \frac{90}{100} \times 28,000$

= · 25,200

Hence

Total claim = 800 + 25,200

= · 26,000

**02.** X : 6 2 10 4 8

Y : 9 11 ? 8 7

Arithmetic means of X and Y series are 6 and 8 respectively . Calculate correlation coefficient

**SOLUTION :**

$$\bar{y} = \frac{\Sigma y}{n} \qquad 8 = \frac{9 + 11 + b + 8 + 7}{5}$$

$$40 = 35 + b \qquad b = 5$$

| x | y | $x-\bar{x}$ | $y-\bar{y}$ | $(x-\bar{x})^2$ | $(y-\bar{y})^2$ | $(x-\bar{x})(y-\bar{y})$ |
|---|---|---|---|---|---|---|
| 6 | 9 | 0 | 1 | 0 | 1 | 0 |
| 2 | 11 | −4 | 3 | 16 | 9 | −12 |
| 10 | 5 | 4 | −3 | 16 | 9 | −12 |
| 4 | 8 | −2 | 0 | 4 | 0 | 0 |
| 8 | 7 | 2 | −1 | 4 | 1 | −2 |
| 30 $\Sigma x$ | 40 $\Sigma y$ | 0 $\Sigma(x-\bar{x})$ | 0 $\Sigma(y-\bar{y})$ | 40 $\Sigma(x-\bar{x})^2$ | 20 $\Sigma(y-\bar{y})^2$ | −26 $\Sigma(x-\bar{x})(y-\bar{y})$ |
| $\bar{x} = 6$ $\bar{y} = 8$ | | | | | | |

$$r = \frac{\Sigma (x-\bar{x}).(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\ \sqrt{\Sigma(y-\bar{y})^2}.}$$

$$r = \frac{-26}{\sqrt{40} \times \sqrt{20}}$$

$$r = \frac{-26}{\sqrt{40 \times 20}}$$

$$r' = \frac{26}{\sqrt{40 \times 20}}$$

taking log on both sides

$$\log r' = \log 26 - \frac{1}{2}\left(\log 40 + \log 20\right)$$

$$\log r' = 1.4150 - \frac{1}{2}[\,1.6021 + 1.3010\,]$$

$$\log r' = 1.4150 - \frac{1}{2}(2.9031)$$

$$\log r' = 1.4150 - 1.4516$$

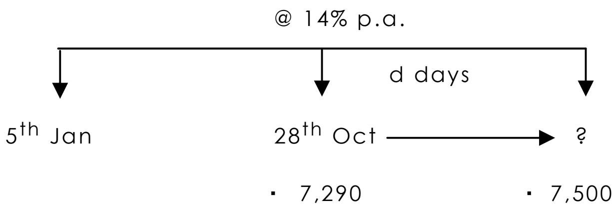
$$\log r' = \bar{1}.9634$$

$$r' = AL(\bar{1}.9634) = 0.9191$$

$$r = -0.9191$$

**03.** a bill of · 7,500 was discounted for · 7290 at a bank on 28th October 2006 . If the rate of interest was 14% p.a. , what is the legal due date

**SOLUTION**




**STEP 1:**

Let Unexpired period = d days

**STEP 2 :**

B.D. = F.V. – C.V.

= 7,500 – 7,290

= · 210

**STEP 3 :**

B.D. = Interest on F.V. for 'd' days @ 14% p.a.

$$210 = \overset{15}{\cancel{7500}} \times \frac{d}{\underset{73}{\cancel{365}}} \times \frac{14}{\cancel{100}}$$

$$d = \frac{210 \times 73}{15 \times 14}$$

$$d = 73 \text{ days}$$

**STEP 4:**

Legal Due date

= 28th Oct + 73 days

| | OCT | | NOV | | DEC | | JAN |
|---|---|---|---|---|---|---|---|
| = | 3 | + | 30 | + | 31 | + | 9 |

= 9th January 2007

**Q6. (A) Attempt any Two of the following (06)**

**01.** The number of complaints which a bank manager receives per day is a Poisson random variable with parameter m = 4 . Find the probability that the manager will receive at most two complaints on any given day ($e^{-4} = 0.0183$)

**SOLUTION**

m = average number of complaints a bank manager receives per day = 4

r.v X ~ P(4)

P( at most two complaints on any given day)

$= P(x \leq 2)$

$= P(0) + P(1) + P(2)$

$= \dfrac{e^{-4}.4^0}{0!} + \dfrac{e^{-4}.4^1}{1!} + \dfrac{e^{-4}.4^2}{2!}$ Using $P(x) = \dfrac{e^{-m}.m^x}{x!}$

$= e^{-4}.\left(\dfrac{1}{1} + \dfrac{4}{1} + \dfrac{16}{2}\right)$

$= 0.0183\,(1 + 4 + 8)$

$= 0.0183(13)$

$= 0.2379$

**02.** Suppose X is a random variable with pdf

$$f(x) = \frac{c}{x} \quad ; \quad 1 < x < 3 \quad ; \quad c > 0$$

Find c & E(X)

i) $$\int_1^3 \frac{c}{x} \, dx = 1$$

$$c \int_1^3 \frac{1}{x} \, dx = 1$$

$$c \left[\log x\right]_1^3 = 1$$

$$c \left[\log 3 - \log 1\right] = 1$$

$$c \log 3 = 1$$

$$c = \frac{1}{\log 3}$$

Hence X is a r.v. with pdf

$$f(x) = \frac{1}{x.\log 3} \quad ; \quad 1 < x < 3$$

ii) $$E(x) = \int_1^3 x.f(x) \, dx$$

$$= \int_1^3 x. \frac{1}{x.\log 3} \, dx$$

$$= \int_1^3 \frac{1}{\log 3} \, dx$$

$$= \left[\frac{x}{\log 3}\right]_1^3$$

$$= \left(\frac{3}{\log 3}\right) - \left(\frac{1}{\log 3}\right) = \frac{2}{\log 3}$$

**03.** In a factory there are six jobs to be performed , each of which should go through machines A and B in the order A – B . Determine the sequence for performing the jobs that would minimize the total elapsed time T . Find T and the idle time on the two machines

| Job | J1 | J2 | J3 | J4 | J5 | J6 |
|---|---|---|---|---|---|---|
| $M_A$ | 1 | 3 | 8 | 5 | 6 | 3 |
| $M_B$ | 5 | 6 | 3 | 2 | 2 | 10 |

**Step 1 : Finding the optimal sequence**

**Min time =** 1 on job $J_1$ on machine $M_1$ . Place the job at the start of the sequence

| | | | | | |
|---|---|---|---|---|---|
| **$J_1$** | | | | | |

**Next min time=** 2 on jobs $J_4$ & $J_5$ on machine $M_B$ . Place the jobs at the end of the sequence randomly

| | | | | | |
|---|---|---|---|---|---|
| **$J_1$** | | | | **$J_4$** | **$J_5$** |

**Placed Randomly**

**Next min time =** 3 on jobs $J_2$ & $J_6$ on machine $M_A$ and on job $J_3$ on machine $M_B$ respectively . Place $J_2$ & $J_6$ at the start next to $J_1$ randomly and $J_3$ at the end next to $J_4$

| | | | | | |
|---|---|---|---|---|---|
| **$J_1$** | **$J_2$** | **$J_6$** | **$J_3$** | **$J_4$** | **$J_5$** |

**Placed Randomly**

**OPTIMAL SEQUENCE**

| | | | | | |
|---|---|---|---|---|---|
| **$J_1$** | **$J_2$** | **$J_6$** | **$J_3$** | **$J_4$** | **$J_5$** |

**Step 2 : Work table**

**According to the optimal sequence**

| **Job** | **$J_1$** | **$J_2$** | **$J_6$** | **$J_3$** | **$J_4$** | **$J_5$** | **total process time** | |
|---|---|---|---|---|---|---|---|---|
| $M_A$ | 1 | 3 | 3 | 8 | 5 | 6 | = | 26 hrs |
| $M_B$ | 5 | 6 | 10 | 3 | 2 | 2 | = | 28 hrs |

**WORK TABLE**

| | MACHINES | | | | Idle time |
|---|---|---|---|---|---|
| | $M_A$ | | $M_B$ | | |
| JOBS | IN | OUT | IN | OUT | on $M_B$ |
| $J_1$ | 0 | 1 | 1 | 6 | 1 |
| $J_2$ | 1 | 4 | 6 | 12 | |
| $J_6$ | 4 | 7 | 12 | 22 | |
| $J_3$ | 7 | 15 | 22 | 25 | |
| $J_4$ | 15 | 20 | 25 | 27 | |
| $J_5$ | 20 | 26 | 27 | 29 | |

**Step 3 :**

**Total elapsed time T = 29 hrs**

**Idle time on $M_A$ = T − (sum of processing time of all 6 jobs on M1)**
**= 29 − 26**
**= 3 hrs**

**Idle time on $M_B$ = T − (sum of processing time of all 6 jobs on M2)**
**= 29 − 28**
**= 1 hr**

**Step 4 : All possible optimal sequences :**

**$J_1 - J_2 - J_6 - J_3 - J_4 - J_5$**

**OR**

**$J_1 - J_6 - J_2 - J_3 - J_4 - J_5$**

**OR**

**$J_1 - J_2 - J_6 - J_3 - J_5 - J_4$**

**OR**

**$J_1 - J_6 - J_2 - J_3 - J_5 - J_4$**

**(B) Attempt any Two of the following** **(08)**

**01.** a pharmaceutical company has four branches , one at each city A , B , C and D . A branch manager is to be appointed one at each city , out of four candidates P , Q , R and S . The monthly business depends upon the city and effectiveness of the branch manager in that city

**MONTHLY BUSINESS (IN LACS)**

| BRANCH MANAGER | CITY: A | B | C | D |
|---|---|---|---|---|
| P | 11 | 11 | 9 | 9 |
| Q | 13 | 16 | 11 | 10 |
| R | 12 | 17 | 13 | 8 |
| S | 16 | 14 | 16 | 12 |

Which manager should be appointed at which city so as to get maximum total monthly business .

Subtracting all the elements in the matrix from its maximum

The matrix can now be solved for 'MINIMUM'

| | | | |
|---|---|---|---|
| 6 | 6 | 8 | 8 |
| 4 | 1 | 6 | 7 |
| 5 | 0 | 4 | 9 |
| 1 | 3 | 1 | 5 |

Reducing the matrix using 'ROW MINIMUM'

| | | | |
|---|---|---|---|
| 0 | 0 | 2 | 2 |
| 3 | 0 | 5 | 6 |
| 5 | 0 | 4 | 9 |
| 0 | 2 | 0 | 4 |

Reducing the matrix using 'COLUMN MINIMUM'

| | | | |
|---|---|---|---|
| 0 | 0 | 2 | 0 |
| 3 | 0 | 5 | 4 |
| 5 | 0 | 4 | 7 |
| 0 | 2 | 0 | 2 |

– Allocation using 'SINGLE ZERO ROW-COLUMN METHOD'
– Allocation incomplete ($3^{rd}$ row unassigned)

| | | | |
|---|---|---|---|
| [0] | ~~0~~ | 2 | ~~0~~ |
| 3 | [0] | 5 | 4 |
| 5 | ~~0~~ | 4 | 7 |
| ~~0~~ | 2 | [0] | 2 |

– Drawing min. no. of lines to cover all '0's

| | | | | |
|---|---|---|---|---|
| | [0] | ~~0~~ | 2 | ~~0~~ |
| √ | 3 | [0] | 5 | 4 |
| √ | 5 | ~~0~~ | 4 | 7 |
| | ~~0~~ | 2 | [0] | 2 |
| | | √ | | |

| | | | | |
|---|---|---|---|---|
| 0 | 3 | 2 | 0 | Revise the matrix |
| 0 | 0 | 2 | 1 | Reducing all the uncovered elements by its minimum and adding the same at the intersection |
| 2 | 0 | 1 | 4 | |
| 0 | 4 | 0 | 2 | |

| | | | |
|---|---|---|---|
| ~~0~~ | 3 | 2 | [0] |
| [0] | ~~0~~ | 2 | 1 |
| 2 | [0] | 1 | 4 |
| ~~0~~ | 4 | [0] | 2 |

– Reallocation using 'SINGLE ZERO ROW-COLUMN METHOD'

– Since all rows contain an 'assigned zero' , the assignment problem is complete

Optimal Assignment : P – D ; Q – A; R – B ; S – C

Maximum business = 9 + 13 + 17 + 16 = 55( lacs)

**02.** Information on vehicles (in thousands) passing through seven different highways during a day (X) and number of accidents reported (Y) is given as

$\Sigma x = 105$ ; $\Sigma y = 409$ ; $\Sigma x^2 = 1681$ ; $\Sigma y^2 = 39350$ ; $\Sigma xy = 8075$ .

Obtain linear regression of Y on X

**SOLUTION**

$$\bar{x} = \frac{\Sigma x}{n} = \frac{105}{7} = 15$$

$$\bar{y} = \frac{\Sigma y}{n} = \frac{409}{7} = 58.43$$

$$byx = \frac{n\Sigma xy - \Sigma x.\Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$$

$$= \frac{7(8075) - (105)(409)}{7(1681) - (105)^2}$$

$$= \frac{56525 - 42945}{11767 - 11025}$$

$$= \frac{13580}{742}$$

$$= 18.30$$

LOG CALC

| |
|---|
| 4.1329 |
| – 2.8704 |
| AL 1.2625 |
| 18.30 |

Equation

$y - \bar{y} = byx\ (x - \bar{x})$

$y - 58.43 = 18.30(x - 15)$

$y - 58.43 = 18.30x - 274.5$

$y = 18.30x - 274.50 + 58.43$

$y = 18.30x - 216.07$

**03.** Minimize $z = 3x_1 + 2x_2$

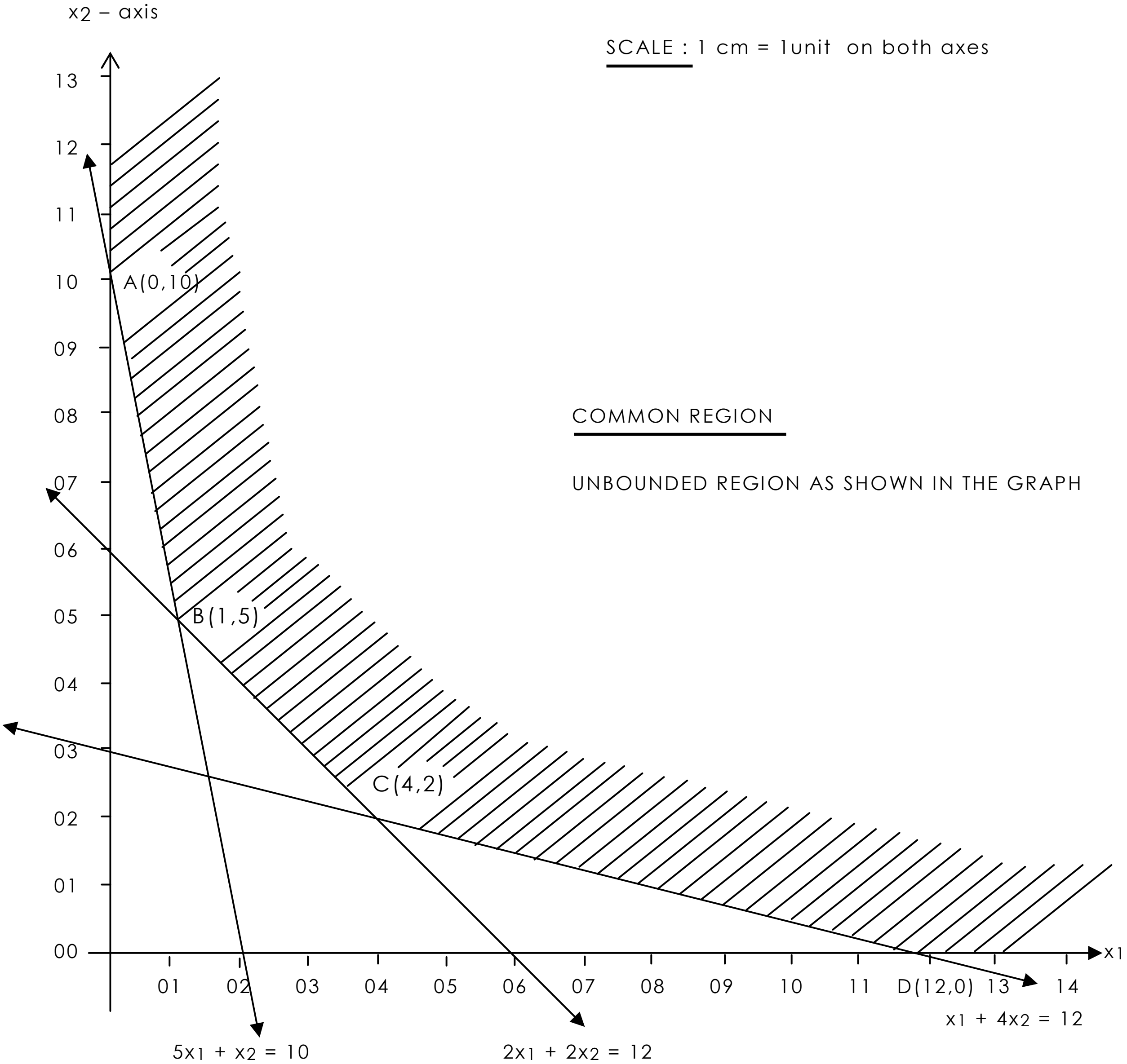
subject to : $5x_1 + x_2 \ge 10$ ; $2x_1 + 2x_2 \ge 12$ ; $x_1 + 4x_2 \ge 12$ ; $x_1, x_2 \ge 0$

STEP 1 :

| | | |
|---|---|---|
| $5x_1 + x_2 \ge 10$ | $5x_1 + x_2 = 10$ | Put (0,0) in $5x_1 + x_2 \ge 10$ |
| | cuts $x_1$ – axis at (2,0) | $0 \ge 10$ |
| | cuts $x_2$ – axis at (0,10) | SS : non – origin side |
| $2x_1 + 2x_2 \ge 12$ | $2x_1 + 2x_2 = 12$ | Put (0,0) in $2x_1 + 2x_2 \ge 12$ |
| | cuts $x_1$ – axis at (6,0) | $0 \ge 12$ |
| | cuts $x_2$ – axis at (0,6) | SS : non – origin side |
| $x_1 + 4x_2 \ge 12$ | $x_1 + 4x_2 = 12$ | Put (0,0) in $x_1 + 4x_2 \ge 12$ |
| | cuts $x_1$ – axis at (12,0) | $0 \ge 12$ |
| | cuts $x_2$ – axis at (0,3) | SS : non – origin side |
| $x_1, x_2 \ge 0$ | | SS : I Quadrant |

STEP 2 :

SCALE : 1 cm = 1unit on both axes




COMMON REGION

UNBOUNDED REGION AS SHOWN IN THE GRAPH

STEP 3 :

| CORNERS | $z = 3x_1 + 2x_2$ | | |
|---|---|---|---|
| A(0,10) | 3(0) + 2(10) | = 0 + 20 | = 20 |
| B(1, 5) | 3(1) + 2(5) | = 3 + 10 | = 13 |
| C(4,2) | 3(4) + 2(2) | = 12 + 4 | = 16 |
| D(12,0) | 3(12) + 2(0) | = 36 + 0 | = 36 |

OPTIMAL SOLUTION : Zmin = 13 at (1,5)

x—x—x—x—x—x—x—x—x—x